STABILITY OF MAGNETIC STARS IN A DECAY FIELD

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ABSTRACT. The question of stability of a magnetic star in a slowly decaying axially symmetric magnetic field is investigated. Poloidal and toroidal fields are considered and the initial magnetic energies evaluated. From the condition of dynamical stability the expression for the critical primitive magnetic fields which can exist in the star is worked out. The magnetic energies and the critical fields for instability are tabulated.

Further the effect of a magnetic field, toroidal as well as poloidal (less than the critical) on the equilibrium of a sphere of constant conductivity is investigated. It is found that the sphere is not an equilibrium form and tends to deform into a spheroidal shape. The effect of a toroidal field is to turn it into a prolate spheroidal shape, whereas in a poloidal field the sphere has a tendency to become an oblate spheroid of small ellipticity. The expressions for the ellipticity of the spheroidal form are worked out in each case.

1. INTRODUCTION

The mechanical equilibrium of a magnetic star has been the subject of investigation during recent years, (Chandrasekhar and Fermi, 1953;) Ferraro, 1954; Gjellestad, 1954; Roberts, 1955; Auluck and Kothari, 1956; Prendergast, 1956; Talwar, 1957). The fluid sphere, in these cases, is assumed to be incompressible, *infinitely conducting*, and situated in vacuum. In this paper, we investigate the problem of the dynamical stability and the equilibrium of an incompressible, gravitating, fluid sphere, in which prevails a slowly decaying magnetic field, which arises due to the Joule heating of a correspondingly decaying current system in the medium of constant electric conductivity σ. The magnetic star is further assumed to be lying at rest in empty space so that the current density j vanishes outside the sphere.

The equations governing a slowly decaying magnetic field in a homogenous sphere of permeability unity, are (using e.m.u.)

$$\text{curl } \boldsymbol{H} = 4\pi \boldsymbol{j}, \quad \text{curl } E = -\frac{\partial \boldsymbol{H}}{\partial t} \qquad \ldots \quad (1)$$

$$\boldsymbol{j} = \sigma E$$

hence

$$\nabla^2 \boldsymbol{H} = 4\pi\sigma \frac{\partial \boldsymbol{H}}{\partial t} \qquad \ldots \quad (2)$$

This equation (2) holds for the interior of the sphere. For regions outside the sphere, the current density j is zero, so that

$$\nabla^2 \boldsymbol{H} = 0 \qquad (3)$$

Elsasser (1946) showed that it is possible to construct three independent solutions of the decaying field equation (2), and the solutions can be written as,

$$\boldsymbol{U} = R\,\Delta\psi,\ \boldsymbol{T} = \text{curl}\,(r\,\psi),\ \boldsymbol{S} = R\ \text{curl}\ \text{curl}\,(\boldsymbol{r}\,\psi) \qquad \ldots \quad (4)$$

where r is the radius vector from the centre,, R is the radius of the sphere and ψ is a scalar generating function satisfying,

$$\nabla^2\psi + k^2\psi = 0 \quad \text{(inside)} \qquad \ldots \quad (5)$$

and

$$\nabla^2\psi = 0 \quad \text{(outside)} \qquad \ldots \quad (6)$$

Here $\qquad k^2 = 4\pi\sigma/\tau,\quad \tau$ being the decay time.

Elsasser calls these functions $\boldsymbol{T}, \boldsymbol{S}, \boldsymbol{U}$ as 'toroidal', 'poloidal' and 'scleoidal' respectively. We shall be concerned in our present work with the toroidal and poloidal functions, and shall be studying the stability of an incompressible sphere, in two cases, one where the field is continuous across the surface of the sphere and the other where the field vanishes on the boundary of the sphere and in the entire space outside it. We shall call these cases as 'Case I' and 'Case II' respectively.

CASE I

2 *Initial magnetic energy, and dynamical instability.*

When equations (5) and (6) are solved with the boundary condition that ψ and $\partial\psi/\partial r$ are continuous at $r = R$, the solutions inside and outside the sphere are given as (Jenson, 1955; Elsasser, 1946)

$$\psi_{ns}{}^{m} = Z_n N_n{}^{m}(R/r)^{\frac{1}{2}} J_{n+\frac{1}{2}}(k_{ns}r) P_n{}^{m}(\mu) e^{im\phi} \qquad (r \leqslant R) \qquad \ldots \quad (7)$$

and

$$\psi_{ns}{}^{m} = Z_n N_n{}^{m}(R/r)^{n+1}\, J_{n+\frac{1}{2}}\,(k_{ns}R) P_n{}^{m}\,(\mu) e^{im\phi} \qquad (r \geqslant R) \quad \ldots \quad (8)$$

where Z_n, $N_n{}^m$ are constants and the characteristic values of k are the zeros of the Bessel function $J_{n-\frac{1}{2}}(k_{ns}R)$.

The equation (4) gives the following components of the functions $\boldsymbol{T}$ and $\boldsymbol{S}$,

$$\boldsymbol{T}_{ns}{}^{m} = \left[\ 0,\ \frac{1}{\sin\theta}\ \frac{\partial\psi_{ns}{}^{m}}{\partial\phi},\ -\ \frac{\partial\psi_{ns}{}^{m}}{\partial\theta}\right]$$

$$\boldsymbol{S}_{ns}{}^{m} = \frac{R}{r}\left[\ n(n+1)\psi_{ns}{}^{m},\ \frac{\partial}{\partial r}\left(r\,\frac{\partial\psi_{ns}{}^{m}}{\partial\theta}\right),\ \frac{1}{\sin\theta}\,\frac{\partial}{\partial r}\left(r\,\frac{\partial}{\partial\phi}\,\psi_{ns}{}^{m}\right)\right] \qquad \ldots \quad (9)$$

We are interested in an axially symmetric field so that the functions T, S do not depend upon the variable ϕ. The components of the internal magnetic field are therefore written as

$$H^{(i)} = \left[C(R/r)^{3/2}n(n+1)J_{n+\frac{1}{2}}(k_{ns}r)P_n(\mu),\ -C\frac{R^{3/2}}{r}\frac{d}{dr}\left\{ r^{\frac{1}{2}}J_{n+\frac{1}{2}}(k_{ns}r)\right\} P_n^1(\mu),\right.$$

$$\left. C(R/r)^{\frac{1}{2}}J_{n+\frac{1}{2}}(k_{ns}r)P_n^1(\mu) \right] \quad \dots \quad (10)$$

The corresponding components of the field outside the sphere are

$$H^{(e)} = [Cn(n+1)(R/r)^{n+2}J_{n+\frac{1}{2}}(k_{ns}R)P_n(\mu),\ Cn(R/r)^{n+2}J_{n+\frac{1}{2}}(k_{ns}R)P_n^1(\mu),$$

$$C(R/r)^{n+1}J_{n+\frac{1}{2}}(k_{ns}r)P_n^1(\mu)] \quad \dots \quad (11)$$

where C is a constant defined in terms of the polar values of the field H_p, as

$$C = \frac{H_p}{n(n+1)J_{n+\frac{1}{2}}(k_{ns}R)} = \frac{(H_p)_0e^{-t/\tau}}{n(n+1)J_{n+\frac{1}{2}}(k_{ns}R)} \quad \dots \quad (12)$$

where $(H_p)_0$ denotes the primitive polar field (at time $t = 0$).

The total magnetic energy, m, is composed of two parts; $m^{(i)}$, the internal magnetic energy and $m^{(e)}$, the external magnetic energy. The internal or external magnetic energy further consists of (i) the contribution $m_p^{(i)\cdot(e)}$ due to the poloidal field (r, and θ-components) and (ii) the contribution $m_T^{(i)\cdot(e)}$ due to the toroidal field (the azimuthal component H_ϕ).

The magnetic energy, m_p, of the configuration due to the poloidal field defined by the components in the r- and θ- directions in equations (10) and (11), is written as

$$m_p = m_p^{(i)}+m_p^{(e)} = \frac{n(n+1)}{4(2n+1)}C^2R^3(k_{ns}R)^2.\ J^2_{n+\frac{1}{2}}(k_{ns}R) \quad \dots \quad (13)$$

Using equations (10) and (11) the contribution to the magnetic energy due to the toroidal field H_ϕ, is written as

$$m_T^{(i)} = \frac{n(n+1)}{4(2n+1)}C^2R^3J^2_{n+\frac{1}{2}}(k_{ns}R) \quad \dots \quad (14)$$

$$m_T^{(e)} = \frac{n(n+1)}{4(2n+1)(2n-1)}C^2R^3J^2_{n+\frac{1}{2}}(k_{ns}R) \quad \dots \quad (15)$$

so that,

$$m_T = m_T^{(i)} + m_T^{(e)}$$

$$= \frac{n(n+1)}{4(2n-1)} C^2 R^3 J^2_{n+\frac{1}{2}}(k_{ns}R) \quad \ldots \quad (16)$$

Thus the total magnetic energy of the configuration is given by

$$m = m_T + m_p$$

$$= \frac{n(n+1)}{4} \; C^2 R^3 J^2_{n+\frac{1}{2}}(k_{ns}R) \left[\frac{k^2_{ns}R^2}{2n+1} + \frac{1}{2n-1} \right] \quad \ldots \quad (17)$$

or, in terms of the primitive polar field

$$m = \frac{(H_p)_0{}^2 R^3}{4n(n+1)} \cdot \left[\frac{k_{ns}{}^2 R^2}{(2n+1)} + \frac{1}{(2n-1)} \right] e^{-2t/\tau}$$

In Table 1, we have tabulated the primitive magnetic energies in units of $H_p{}^2 R^3$.

The gravitational potential energy of a homogenous sphere is given by

$$\Omega = -\frac{3}{5} \frac{GM^2}{R} \quad \ldots \quad (18)$$

For dynamical stability the magnetic energy (eq 17) should be less than the gravitational energy (eq. 18); thus the critical magnetic field parameter C^* is given by

$$C^* = \left[\frac{12}{5} \frac{(2n-1)(2n+1)}{\{n(n+1)\}\{(2n-1)k_{ns}{}^2R^2 + (2n+1)\}} \right]^{1/2} \frac{G^{1/2}M}{R^2 J_{n+1/2}(k_{ns}R)} \quad \ldots \quad (19)$$

The condition for dynamical stability can be alternatively written in terms of the primitive polar field $(H_p)_0$, as

$$(H_p)_0 < \left[\frac{12}{5} \frac{n(n+1)(2n-1)(2n+1)}{(2n-1)k_{ns}{}^2R^2 + (2n+1)} \right]^{1/2} \frac{G^{1/2}M}{R^2} \quad \ldots \quad (20)$$

Here $k_{ns}R$ denote the zeros of the Bessel function $J_{n-\frac{1}{2}}(k_{ns}R)$. Table. II is derived from the equation (20) for a star having solar dimension and mass.

The equation (19) defines the critical maximum magnetic field which can exist with a sphere with constant electrical conductivity. In the following section (3) we investigate the effect of a decaying magnetic field less than the critical field (the field for dynamical instability to set in) on the mechanical equilibrium of star.

TABLE I

The magnetic energies in some modes for a sphere of uniform conductivity

n	s	Magnetic energy in units of $H_p{}^2R^3$	n	s	Magnetic energy in units of $H_p{}^2R^3$
1	1	0.536	3	1	0.110
1	2	1 770	3	2	0.251
1	3	3.827	3	3	0.458
1	4	6.707	3	4	0.738
1	5	10.410	3	5	1.056
2	1	0.182			
2	2	0 620			
2	3	1.093			
2	4	1.726			
2	5	2.519			

TABLE II

Critical primitive polar fields for instability for a magnetic star of solar dimensions

n	s	$H_p \times 10^{-7}$ gauss	n	s	$H_p \times 10^{-7}$ gauss
1	1	5.37	2	3	3.94
1	2	2.94	2	4	3.06
1	3	1.98	2	5	2.49
1	4	1 50	3	1	11.70
1	5	1 2	3	2	7.56
			3	3	5.72
2	1	9 06	3	4	4.38
2	2	5.50	3	5	3.77

3. *Equilibrium of a sphere under poloidal and toroidal fields (eq. 10, 11).*

For the purpose of investigating whether a sphere is an equilibrium form under the magnetic field defined by equations (10) and (11), we give the sphere a virtual displacement which changes the boundary $r = R$ of the sphere into one defined

$$r(\mu) = R+\epsilon P_l(\mu) \qquad \dots \quad (21)$$

and calculate the change in the total energy of the configuration (magnetic plus gravitational). If the change in the total energy of the configuration vanishes, to the first order in ϵ for all values of l in equation (21), (for all modes of P_l – deformation), we can then conclude that a sphere is a form of equilibrium. If, on the other hand, it vanishes for only some particular values of l, the maximum we can conclude is that the sphere is a form of equilibrium only for these particular modes of P_l-deformation and not *in general* so that the sphere is not a form of *true* equilibrium.

In order to evaluate the change in the total energy of the configuration, we need expressions for the change in gravitational potential energy, $\Delta\Omega$, and for the change in the magnetic energy, Δm, due to the P_e-deformation (21). The former is calculated by Chandrasekhar and Fermi (1953) and is given by,

$$\Delta\,\Omega_l = \frac{3(l-1)}{(2l+1)^2}\ \epsilon^2 \frac{GM^2}{R^3} \tag{22}$$

which is of second order in ϵ and is always positive. If Δm also turns out to be of second order in ϵ and positive, then the sphere is a stable form of equilibrium. We shall find that the expression for the change in magnetic energy is non-vanishing, up to first order in ϵ, for a P_2-deformation, and is zero for all other deformations ($l\neq 2$). Thus we can conclude that the magnetic star (sphere) is unstable for a P_2-deformation, and would tend to become spheroidal although for any other deformation, we cannot decide about the stability question unless we go upto second order in ϵ. This, however, is beyond the scope of the present work.

The change in the magnetic energy, Δm, as a consequence of the P_l-deformation, can be written as,

$$\Delta m = -\int \boldsymbol{\xi}\cdot(\boldsymbol{J}\times\boldsymbol{H})d\tau \tag{23}$$

The result shall be true up to first order in ϵ when the volume integral is taken over the undisturbed body and $\boldsymbol{J}$ and $\boldsymbol{H}$ denote the undisturbed current density and field.

In equation (23), $\boldsymbol{\xi}$ denotes the displacement corresponding to deformation (21), and is written as (Chandrasekhar and Fermi, 1953),

$$\xi_r = \epsilon(r/R)^{l-1}P_e(\mu),\ \ \xi_\theta = -\epsilon/l(r/R)^{l-1}P_l^1(\mu) \qquad \ldots \quad (24)$$

The current density $\boldsymbol{J}$ in zero outside the sphere, whereas in the interior of the sphere it is given by,

$$4\pi\boldsymbol{J} = \text{curl}\ \boldsymbol{H} \qquad \ldots \quad (25)$$

so that we can write, using equation (10),

$$4\pi j_r = \frac{CR^{\frac{1}{2}}}{r^{3/2}}\ J_{n+\frac{1}{2}}(k_{ns}r)n(n+1)\,P_n(\mu)$$

$$4\pi j_\theta = -\frac{CR^{1/2}}{r^{3/2}}\ [\,-nJ_{n+\frac{1}{2}}(k_{ns}r)+k_{ns}rJ_{n-\frac{1}{2}}(k_{ns}r)]\,P_n^1(\mu) \tag{26}$$

and

$$4\pi j_\phi = \frac{Ck_{ns}^2}{r^{1/2}} R^{3/2} J_{n+\frac{1}{2}}(k_{ns}r)(1-\mu^2)^{\frac{1}{2}} P_n'(\mu)$$

The Lorenty force is written as,

$$F = [j\times H] = [j_\theta H_\phi - j_\phi H_\theta, j_\phi H_r - j_r H_\phi, j_r H_\theta - j_\theta H_r] \quad \dots \quad (27)$$

Thus the expression (23) can be rewritten as,

$$\Delta m = -\int_\tau (\xi_r F_r + \xi_\theta F_\theta) d\tau = -\int_\tau [\xi_r (j_\theta H_\phi - j_\phi H_\theta) + \xi_\theta (j_\phi H_r - j_r H_\phi)] d\tau$$

$$= \Delta m_p + \Delta m_T \quad \dots \quad (28)$$

where

$$\Delta m_p = -\int [\xi_r(-j_\phi H_\theta) + \xi_\theta (j_\phi H_r)] d\tau \quad \dots \quad (29)$$

and

$$\Delta m_T = -\int [\xi_r (j_\theta H_\phi) + \xi_\theta(-j_r H_\phi)]\ d\tau \quad \dots \quad (30)$$

The expression (29) gives the change in the magnetic energy of a sphere due to a P_l deformation, when the magnetic field assumed is purely poloidal (r, θ components), whereas equation (30) gives the change in magnetic energy for the same deformation when the magnetic field is purely toroidal in character.

The equation (29) can be written as, making use of the equations (10), (24), and (26),

$$\Delta m_p = -\frac{\epsilon C^2 R^3 k_{ns}^2}{2R^{l-1}} \int_{-1}^{+1}\int_0^R \Big[[P_n^1(\mu)]^2 P_l(\mu) r^{l-1} J_{n+\frac{1}{2}}(k_{ns}r) \times \{-nJ_{n+\frac{1}{2}}(k_{ns}r) +$$

$$k_{ns}rJ_{n-\frac{1}{2}}(k_{ns}r)\} - \frac{n(n+1)}{l} r^{l-1} [J_{n+\frac{1}{2}}(k_{ns})r]^2 P_l^1(\mu) P_n^1(\mu) P_n(\mu)\Big] dr d\mu \quad \dots \quad (31)$$

Since the integration over μ in equation (31) vanishes for $(2n+l) =$ odd i.e. $l =$ odd, we conclude that

$$\Delta m_p = 0 \quad \text{for all } n, \quad \text{but } l = \text{odd} \quad \dots \quad (32)$$

Further, it can be easily shown that for $n = 1$ the change in the poloidal magnetic energy, Δm_p, vanishes for all l, except $l = 2$, which corresponds to a

P_2-deformation. For a P_2-deformation, the change, Δm_p, in poloidalm agnetic energy is of first order in ϵ, and is given by

$$\Delta m_p = -\frac{\epsilon C^2 R^2 k_{ns}^2}{2} \left[\frac{2n(n+1)\{n(n+1)-3\}}{(2n-1)(2n+1)(2n+3)} \times \int_0^R r J_{n+\frac{1}{2}}\{-nJ_{n+\frac{1}{2}}+k_{ns} r J_{n-\frac{1}{2}}\} dr \right.$$

$$\left. -\frac{n(n+1)}{2} \int_0^R \int_{-1}^{+1} P_2^1(\mu) P_n^1(\mu) P_n(\mu) r [J_{n+\frac{1}{2}}(k_{ns}r)]^2 dr\, d\mu \right] \quad \ldots \quad (33)$$

For the simplest decay field, corresponding to $n=1$, we have

$$\Delta m_{1s} = 0 \text{ for } l \neq 2$$

and

$$\Delta m_{\substack{1s \\ (l=2)}} = \frac{2}{15} C^2 \epsilon R^2 \left[(k_{1s}R)^2 J_{3/2}(k_{1s}R) + \frac{1}{\pi} \left\{ k_{1s}R - \frac{3}{4} \sin 2k_{1s}R \right.\right.$$

$$\left.\left. + \frac{k_{1s}R}{2} \cos 2k_{1s}R \right\} \right] \quad \ldots \quad (34)$$

Since $k_{1s}R$ are zeros of the function $J_{\frac{1}{2}}(k_{1s}R)$, the above expression for the change in the poloidal magnetic energy, for a P_2—deformation, can be put down as

$$[\Delta m_{1s}]_p = \frac{7}{15} \epsilon s C^2 R^2 = \frac{7}{120} \pi^2 s^2 c (H_p)_0^2 R^2 e^{-2t/\tau} \quad \ldots \quad (35)$$

Thus we see that in contrast to $\Delta\Omega$, (equation 22), which is of second order in ϵ and always positive, the change, Δm_p, in the poloidal magnetic energy has a contribution even up to first order in ϵ for a P_2-deformation. Hence we conclude that a magnetic sphere of constant conductivity with poloidal field given by equation (10), is not an equilibrium form and would tend to an oblate spheroidal form for a P_2-deformation. The extent of the collapse towards the oblate shape in the simplest decay field, can be found immediately by minimising the total change $(\Delta\Omega+\Delta m_{1p})$ for a P_2-deformation . This leads us to the expression

$$(\epsilon/R)_{1s} = -\frac{35}{144} s^2 \pi^2 \frac{(H_p)_0^2 R^4}{GM^2} e^{-2t/\tau} \quad \ldots \quad (36)$$

The other part, Δm_T, in equation (28), which represents the change in the toroidal magnetic energy, can be evaluated, with the help of equation (10), (24) and (26). The results are expressed as,

$$\Delta m_T = 0 \quad \text{for all } n, \quad \text{but } l = \text{odd}$$

$$= 0 \quad \text{for } n = 1, \quad \text{but } l \neq 2 \quad \ldots \quad (37)$$

For a P_2-deformation, ($l = 2$), the change in the magnetic energy due to the toroidal component, Δm_T, is non-vanishing and is given by, for the simplest decay field,

$$[\Delta m_{1s}]_T = -\frac{4}{15}\frac{C^2\epsilon R^2}{\pi^2 s} \quad \ldots \quad (38)$$

Thus the change in the total magnetic energy, Δm, due to a P_l-deformation of a sphere, with both toroidal and poloidal decaying fields, is

$$\begin{aligned} \Delta m &= \Delta m_T + \Delta m_p \\ &= 0 \quad \text{for all } n, \quad \text{but } l = \text{odd} \\ &= 0 \quad \text{for } n = 1, \quad \text{but } l \neq 2 \end{aligned} \quad \ldots \quad (39)$$

and

$$[\Delta_1 m_s]_{r=2} = \frac{7}{15}\epsilon s C^2 R^2 \left(1 - \frac{4}{7\pi^2 s^2}\right) \quad \ldots \quad (40)$$

Since s is a positive integer, Δm, does not vanish for any value of s. Thus we see that the change, Δm, in the magnetic energy is of first order in ϵ, whereas the change $\Delta\Omega$ is of second order in ϵ and is always positive. We conclude, therefore, that with both toroidal and poloidal field (eq.10,11) the sphere is not in real equilibrium and would deform into a spheroidal shape of ellipticity ϵ/R, given by

$$\begin{aligned} (\epsilon/R)_{1s} &= -\frac{35}{144}\frac{s^2\pi^2 R^4}{GM^2}(H_p)_0^2 e^{-2t}\left(1 - \frac{4}{7\pi^2 s^2}\right) \quad \ldots \quad (41) \\ &= -3.5(H_p/H_p^{\times})_0^2\, e^{-2/\tau}\left[1 - \frac{4}{7\pi^2 s^2}\right] \end{aligned}$$

in terms of the critical primitive polar field.

If only toroidal field were present, its effect is to deform the sphere into a prolate spheroid of ellipticity,

$$(\epsilon/R)_{1s} = \frac{10}{9s\pi^2}\frac{C^2R^4}{GM^2} \quad \ldots \quad (42)$$

CASE II

4: Stability of a sphere with poloidal and and toroidal field, vanishing at surface.

When the internal field vanishes on the surface, $r = R$, of the sphere and in the space outside it, the calculations differ from Case I in that the characteristic values l_{ns} are zeros of the Bessel function $J_{n+\frac{1}{2}}(l_{ns}R)$.

The field components can then be written as,

$$H = \Big[C'n(n+1)(R/r)^{3/2}J_{n+\frac{1}{2}}(l_{ns}r)P_n(\mu),\ -\frac{C'R^{3/2}}{r}\frac{d}{dr}$$

$$\{r^{\frac{1}{2}}J_{n+\frac{1}{2}}(l_{ns}r)\}P_n^1(\mu), C'(R/r)^{\frac{1}{2}}J_{n+\frac{1}{2}}(l_{ns}r)P_n^1(\mu) \Big] \quad \ldots \quad (43)$$

The corresponding magnetic energy is

$$m = \frac{n(n+1)}{4(2n+1)} C'^2R^3[1+l^2_{ns}R^2], \quad \ldots \quad (44)$$

so that the sphere shall be dynamically stable for values of the field parameter C', less than the critical defined by,

$$C'^{\times} = \left\{ \frac{12(2n+1)^{\frac{1}{2}}}{5n(n+1)} \right\} \cdot \frac{G^{\frac{1}{2}}M}{R^2} \frac{1}{(1+l^2_{ns}R^2)^{\frac{1}{2}}J_{n-\frac{1}{2}}(l_{ns}R)} \quad \ldots \quad (45)$$

Again for fields less than this critical, the change $\Delta\eta$ in the magnetic energy, due to a P_l -deformation of the sphere, is written as,

$$\Delta m = 0 \qquad \text{for all } n, \quad \text{but} \quad l = \text{odd} \quad \ldots \quad (46)$$

and

$$\Delta m = 0 \qquad \text{for } n = 1, \quad \text{but } l \neq 2$$

For a P_2-deformation, the change Δm_s in magnetic energy is non-vanishing and is given by, (for $n = 1$),

$$\Delta m_{1s} = (\Delta m_{1s})_T + (\Delta m_{1s})_p \quad \ldots \quad (47)$$

The change, $[\Delta m_{1s}]_p$ in the poloidal magnetic energy can be easily worked out and is given by,

$$[\Delta m_{1s}]_p = \frac{2}{15} C'^2\epsilon R^2 \left[J^2_{\frac{1}{2}}(l_{1s}R) + \frac{l_{1s}R}{2\pi\{1+l^2_{1s}R^2\}} \right] l^2_{1s}R^2 \quad \ldots \quad (48)$$

and the corresponding expression for the change $[\Delta m_{1s}]_T$ in the toroidal energy is,

$$[\Delta m_{1s}]_T = -\frac{2}{15} C'^2\epsilon R^2 \left[\frac{J^2_{\frac{1}{2}}(l_{1s}R)}{4} + \frac{l_{1s}R}{2\pi\{1+l^2_{1s}R^2\}} \right] \quad \ldots \quad (49)$$

so that the total change in magnetic energy due to a P_2-deformation of a sphere with simplest decay field (poloidal+toroidal) becomes

$$\Delta m_{1s} = \frac{2}{15} C'^2\epsilon R^2 \left[J^2_{\frac{1}{2}}(l_{1s}R)\{l^2_{1s}R^2-\tfrac{1}{4}\} + \frac{l_{1s}R\{l^2_{1s}R^2-1\}}{2\pi\{l^2_{1s}R^2+1\}} \right] \quad \ldots \quad (50)$$

Thus we see that the sphere is not in true equilibrium with the decay field (poloidal+toroidal) vanishing at the surface and in the space outside, since the change in magnetic energy is non-vanishing for P_2-deformation. The sphere shall deform to a spheroidal shape, and the extent of deformation is easily written as (for $n = 1$)

$$(\epsilon / R)_{1s} = -\frac{5}{9}\frac{C'^2R^4}{GM^2}\left(J^2_{\frac{3}{2}}(l_{1s}R)\left\{ l_{1s}^2R^2 - \frac{1}{4}\right\} + \frac{l_{1s}R\{l^2_{1s}R^2-1\}}{2\pi\{l_{1s}^2R^2+1\}}\right) \quad \ldots \quad (51)$$

If the field prevalent in the sphere is purely toroidal, the sphere will tend to a prolate spheroidal shape of ellipticity,

$$(\epsilon/R)_{1s} = \frac{5}{9}\frac{C'^2R^4}{GM^2}\left[\frac{J^2_{\frac{3}{2}}(l_{1s}R)}{4} + \frac{l_{1s}R}{2\pi\{1+l^2_{1s}R^2\}}\right] \quad \ldots \quad (52)$$

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REFERENCES

Auluck, F. C. and Kothari, D. S., 1956, *Proc. Nat. Inst. Sci* (In press).

Chandrasekhar, S. and Fermi, E., 1953, *Ap. J.* **118**, 116.

Elsasser, W M., 1946, *Phy Rev.*, **69**, 106

Ferraro, V C. A., 1954, *Ap. J.* **119**, 407.

Gjellestad, G., 1954, *Ap J* **120**, 172.

Jenson, E., 1955, *Ap. J.* (Supplement No. 16), 141.

Lust, R. and Schluter, A., 1954, *ZAp.* **34**, 263

Prendergast, K H., 1956, *Ap. J.* **123**.

Roberts, P. H , 1956, *Ap. J.* **122**, 508.

Stratton, J. A., 1941, Electromagnetic Theory (McGraw Hill Book Co., Inc., New York) Chap 7.

Talwar, S. P , 1957, *Zeits. f. Ap.* **42**, 42.